Measured Quantum Dynamics of a Trapped Ion

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The measurement process is taken into account in the dynamics of trapped ions prepared in nonclassical motional states. The induced decoherence is shown to manifest itself both in the inhibition of the internal population dynamics and in a damping of the vibrational motion without classical counterpart. Quantitative comparison with present experimental capabilities is discussed, leading to a proposal for the verification of the predicted effects.

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Quantum decoherence is commonly understood as the decay of the off-diagonal density matrix elements responsible for the nonclassical properties of superposition states. Its key role in the transition from quantum to classical physics has been lately recognized, underlying the reason we are prevented from everyday observation of quantum features in the macroscopic realm [1,2]. Decoherence is usually conspiring against stable preparation of quantum states through different mechanisms. Despite of the fact that some of them, for instance of thermal or instrumental nature, can be quenched by proper technological developments, an *ultimate* source remains due to the coupling between the observed system and the meter. This coupling has in principle to be taken into account whenever physical information is extracted from the system.

Several proposals have been discussed so far for experimentally testing the issue of decoherence, among these the ones employing superposition states in molecules or crystals [3], superconducting rings [4], nonlinear Kerr media [5] and cavity QED setups [6,7]. Recently, a further scenario has been disclosed succeeding in the generation and detection of nonclassical motional states of single trapped and cooled ions [8], also opening the way to the creation of mesoscopic Schrödinger cat states and the controlled study of their subsequent death [9]. The possibility to manipulate the internal and external state of the ion by means of suitably arranged laser beams, after cooling down to zero-point vibrational energy [10], and the extremely low dissipation attainable on the vibrational motion itself, have been crucial in determining the success of this technique.

It is the purpose of this paper to investigate the decoherence introduced by a continuous measurement process on the dynamics of a trapped and laser-irradiated ion. It turns out that peculiar quantum mechanical effects are in principle detectable in the evolution of both electronic and motional degrees of freedom of the trapped ion.

The starting point of our analysis is the Lindblad equation for the density operator $\hat{\rho}(t)$ of a system subjected to a continuous measurement of the generic observable $\hat{A}$ [11–13],

$$\frac{d\hat{\rho}(t)}{dt} = -\frac{i}{\hbar}[\hat{H}_0 + \hat{H}_{int}(t), \hat{\rho}(t)] - \frac{\kappa}{2}[\hat{A}, [\hat{A}, \hat{\rho}(t)]] \, . \tag{1}$$

The first contribution in the right-hand side of (1) describes the dynamics of the unmeasured system. The Hamiltonian operator $\hat{H}_0$ accounts for the free motion of the vibrational and electronic degrees of freedom of the ion,

$$\hat{H}_0 = \hat{H}^{cm} + \hat{H}^{el} = \hbar\omega \, \hat{a}^\dagger \hat{a} + \hbar\omega_{21} \, \hat{\sigma}_z \, , \tag{2}$$

being ω the frequency of the harmonic trap, hereafter assumed to be much larger than the atomic radiative linewidth (strong confinement limit, $\omega \gg \Gamma$), ω_{21} the transition frequency of the two-level system, $\hat{a}$ and $\hat{\sigma}_z$ respectively the annihilation operator of the 1D boson mode and the atomic pseudo-spin $\hat{z}$-operator. Coupling between internal and translational dynamics is achieved through a nonlinear multiquantum Jaynes-Cummings Model (JCM) interaction, which in rotating-wave approximation is written as [14,15]

$$\hat{H}_{int}(t) = \frac{\hbar\Omega_0}{2} \, \mathrm{e}^{i\omega_L t} \cos[\eta(\hat{a} + \hat{a}^\dagger) + \varphi] \, \hat{\sigma}_- + \mathrm{H.c.} \, , \tag{3}$$

where ω_L is the laser operating frequency, Ω_0 the fundamental Rabi frequency, $\eta = (\omega_L/c)(\hbar/2m\omega)^{1/2}$ the Lamb-Dicke parameter related to the standard quantum limit for the localization of the ion and $\hat{\sigma}_-$ the electronic lowering operator. In Eq. (3) a standing-wave laser field is considered, the phase φ determining the position of the trap potential with respect to the wave. If, in particular, the ion is trapped at the node of the light field ($\varphi = \pm\pi/2$)

and $\eta \ll 1$ (Lamb-Dicke limit), the well-known linear one-quantum JCM or anti-JCM operators $\eta(\hat{a}\hat{\sigma}_+ + \hat{a}^\dagger\hat{\sigma}_-)$, $\eta(\hat{a}^\dagger\hat{\sigma}_+ + \hat{a}\hat{\sigma}_-)$ may be recovered [16–18]. The second contribution in Eq. (1) is responsible for the opening of the dynamics due to the effect of the measurement. The parameter κ, supposed to be time-independent, expresses the strength of the coupling to the measuring apparatus [13]. The choice of the measured observable $\hat{A}$ clearly depends upon the specific experimental procedure. If the occupancy of the internal ground state $\downarrow$ is monitored, for instance by collecting the fluorescence light emitted after stimulated transitions to a third auxiliary level as done in [8], the appropriate operator in (1) is given in the pseudo-spin formalism by $\hat{A} = \hat{\sigma}_-\hat{\sigma}_+$. Any other admissible observable of the system could be analyzed in principle.

It is convenient to project Eq. (1) in the representation defined by the eigenkets of $\hat{H}_0$, $|S, n\rangle$, $S = \downarrow, \uparrow$, $n = 0, \ldots, \infty$. The analysis is greatly simplified if, as in practical conditions [8,10], the ion is in the low excitation regime ($\omega \gg \Omega_0$) and the laser is tuned to the k-th vibrational sideband, $\omega_L = \omega_{21} \pm k\omega$, with $+$ ($-$) corresponding to blue (red) respectively. Thus transitions involving the exchange of k vibrational quanta are resonantly enhanced, while the off-resonant couplings rapidly oscillating at frequency ω can be disregarded, the so-called JCM approximation [14,15]. By choosing, without loss of generality, a k-th blue resonance, transitions between states $|\downarrow, n\rangle$, $|\uparrow, n+k\rangle$ are driven and the master equation (1) leads to the following closed set of equations

$$\begin{cases} \dot{\rho}_{\downarrow n \downarrow m}(t) &= -i\omega(n-m)\rho_{\downarrow n\downarrow m}(t) - i/2\left[\Omega_{nn+k}\rho_{\uparrow n+k \downarrow m}(t)\mathrm{e}^{i\omega_L t} - \Omega_{mm+k}\rho_{\downarrow n \uparrow m+k}(t)\mathrm{e}^{-i\omega_L t}\right] , \\ \dot{\rho}_{\uparrow n+k \uparrow m+k}(t) &= -i\omega(n-m)\rho_{\uparrow n+k\uparrow m+k}(t) - i/2\left[\Omega_{nn+k}\rho_{\downarrow n \uparrow m+k}(t)\mathrm{e}^{-i\omega_L t} - \Omega_{mm+k}\rho_{\uparrow n+k \downarrow m}(t)\mathrm{e}^{i\omega_L t}\right] , \\ \dot{\rho}_{\downarrow n \uparrow m+k}(t) &= \left[-i\left(\omega(n-m) - \omega_{21}\right) - \kappa/2\right]\rho_{\downarrow n\uparrow m+k}(t) - i/2\left[\Omega_{nn+k}\rho_{\uparrow n+k \uparrow m+k}(t) - \Omega_{mm+k}\rho_{\downarrow n \downarrow m}(t)\right]\mathrm{e}^{i\omega_L t} , \\ \dot{\rho}_{\uparrow n+k \downarrow m}(t) &= \left[-i\left(\omega(n-m) + \omega_{21}\right) - \kappa/2\right]\rho_{\uparrow n+k\downarrow m}(t) - i/2\left[\Omega_{nn+k}\rho_{\downarrow n \downarrow m}(t) - \Omega_{mm+k}\rho_{\uparrow n+k \uparrow m+k}(t)\right]\mathrm{e}^{-i\omega_L t} , \end{cases} \tag{4}$$

where, as customary, the nonlinear k-quantum Rabi frequencies have been introduced [15],

$$\Omega_{nn+k} = \Omega_0 \langle n|\, \cos[\eta(\hat{a} + \hat{a}^\dagger) + \varphi]\, |n+k\rangle \, . \tag{5}$$

Eqs. (4) can be analitically solved through standard techniques, the details of which will be reported in a future extended paper. Two special cases can be easily handled. When $\kappa = 0$ the solution of Eqs. (4) is straightforwardly obtained by solving the Schrödinger equation for the closed system; for $\kappa > 0$ and $n = m$ a two-level measured evolution is instead recognizable within each Jaynes-Cummings manifold $\{|\downarrow, n\rangle, |\uparrow, n+k\rangle\}$. In analogy to the situation already investigated in [13], a Rabi-like oscillatory behavior with frequency Ω_{nn+k} or a Zeno-like dynamically frozen regime are then expected for values of κ respectively smaller or larger than the critical one $\kappa^{crit}_{nn+k} = 4\Omega_{nn+k}$. Starting from the general solution of Eqs. (4), the properties of the internal and vibrational motions can be studied by considering the reduced density matrices

$$\sigma_{SS'}(t) = \sum_{n=0}^{\infty} \rho_{Sn,S'n}(t) \, , \qquad \rho^{cm}_{nm}(t) = \sum_{S=\downarrow,\uparrow} \rho_{Sn,Sm}(t) \, . \tag{6}$$

As in the experiment reported by Meekhof *et al.* [8], we will assume that at initial time no entanglement between internal and translational degrees of freedom is present and only the lower electronic level is populated, *i.e.* $\rho_{Sn,S'm}(0) = \sigma_{SS'}(0)\,\rho^{cm}_{nm}(0) = \delta_{SS'}\delta_{S\downarrow}\,\rho^{cm}_{nm}(0)$.

Let us first focus on the effect of the measurement on the internal subdynamics. The measured occupancy of level $\downarrow$, $P_\downarrow(t) = \mathrm{Tr}(\hat{\sigma}(t)\hat{\sigma}_-\hat{\sigma}_+) = \sigma_{\downarrow\downarrow}(t)$ is found to be

$$P_\downarrow(t) = \frac{1}{2}\left\{1 + \mathrm{e}^{-(1/4)\kappa t} \sum_{n=0}^{\infty} \rho^{cm}_{nn}(0)\left[\cos(w_{nn}t) + \frac{\kappa}{4w_{nn}}\sin(w_{nn}t)\right]\right\} , \tag{7}$$

where the diagonal center-of-mass matrix elements characterize the number state distribution of the vibrational motion and the following conventions are introduced, for future reference, for the frequencies

$$w_{nm} = \sqrt{\left(\frac{\Omega_{nn+k} + \Omega_{mm+k}}{2}\right)^2 - \frac{\kappa^2}{16}} \, , \qquad u_{nm} = \sqrt{\left(\frac{\Omega_{nn+k} - \Omega_{mm+k}}{2}\right)^2 - \frac{\kappa^2}{16}} \, . \tag{8}$$

Eq. (7) has to be compared with the analysis presented in Ref. [8]. Apart from a factor 2 entering the definition of the multiquantum frequencies (5), it is worth to emphasize that the decay rate of the signal (7), determined by the value of κ, is independent upon the vibrational quantum number, corroborating a different origin of the n-increasing decoherence instead observed in [8] and there ascribed to laser intensity noise and trap drive instabilities. In the simplest case of an initial Fock state, $\rho^{cm}_{nn}(0) = \delta_{n\overline{n}}$, Eq. (7) simplifies to

$$P_{\downarrow}^{Fock}(t) = \frac{1}{2}\left\{1 + \mathrm{e}^{-(1/4)\kappa t}\left[\cos(w_{\overline{n}\,\overline{n}}t) + \frac{\kappa}{4w_{\overline{n}\,\overline{n}}}\sin(w_{\overline{n}\,\overline{n}}t)\right]\right\}. \tag{9}$$

In the Zeno-like regime, $\kappa > \kappa^{crit}_{\overline{n}\,\overline{n}}$, the frequency $w_{\overline{n}\,\overline{n}}$ becomes imaginary and an almost complete inhibition of the evolution occurs after a transient of the order κ^{-1}. Moreover, according to (9), even in the opposite weak coupling limit $0 < \kappa \ll \kappa^{crit}_{\overline{n}\,\overline{n}+k}$ the measurement induced decoherence appears through three distinct signatures: an exponential decay of the signal amplitude, a frequency shift of the pseudo-oscillation and the presence of a sinusoidal term. More elaborated examples are available by changing the initial vibronic distribution in (7), as discussed in [19] with emphasis on the collapses and revivals arising from a coherent initial state.

Until now an effect of the measurement has been identified on the dynamics of the internal degree of freedom, a fact which is quite expected since the latter is *directly* related to the observed quantity $\hat{A}$. However, the back-action of the measurement on the evolution of the coupled vibrational motion has likewise to be examined. The motional degree of freedom also feels the measuring apparatus, originating *indirect* decoherentization of the reduced density matrix $\rho^{cm}_{nm}(t)$ introduced in (6) and therefore modifying the average values of any center-of-mass observable, $\langle\hat{\mathcal{O}}^{cm}(t)\rangle = \mathrm{Tr}(\hat{\rho}^{cm}(t)\hat{\mathcal{O}}^{cm})$. We restrict ourselves here to two representative cases, center-of-mass position and energy. They can be shown to evolve respectively as

$$\begin{aligned}\langle\hat{x}^{cm}(t)\rangle = \sqrt{\frac{\hbar}{2m\omega}}\mathrm{e}^{-(1/4)\kappa t}\sum_{n=0}^{\infty}\Re\mathrm{e}\Big\{\mathrm{e}^{i\omega t}\rho^{cm}_{nn+1}(0)\Big[(\sqrt{n+k+1}+\sqrt{n+1})\Big[\cos(u_{nn+1}t) + \frac{\kappa}{4u_{nn+1}}\sin(u_{nn+1}t)\Big]\\ - (\sqrt{n+k+1}-\sqrt{n+1})\Big[\cos(w_{nn+1}t) + \frac{\kappa}{4w_{nn+1}}\sin(w_{nn+1}t)\Big]\Big]\Big\},\end{aligned} \tag{10}$$

$$\langle\hat{H}^{cm}(t)\rangle = \hbar\omega\left(\overline{n} + \frac{1}{2} + \frac{k}{2}\right) - \frac{\hbar\omega}{2}\,k\,\mathrm{e}^{-(1/4)\kappa t}\sum_{n=0}^{\infty}\rho^{cm}_{nn}(0)\Big[\cos(w_{nn}t) + \frac{\kappa}{4w_{nn}}\sin(w_{nn}t)\Big], \tag{11}$$

being u_{nn+1}, w_{nn+1} and w_{nn} previously defined in (8). Eqs. (10) and (11) contain the main result of this paper, implying that the average position and energy exponentially relaxe to a constant value with a rate $\kappa/4$, a *quantum damping* purely due to the measurement process, as already introduced for the case of a single degree of freedom [20]. More generally, quantum damping can be proven to occur for any observable $\hat{\mathcal{O}}^{cm}$, leading to the asymptotic behavior

$$\lim_{t\to\infty}\langle\hat{\mathcal{O}}^{cm}(t)\rangle = \frac{1}{2}\sum_{n=0}^{\infty}\Big(\langle n|\hat{\mathcal{O}}^{cm}|n\rangle + \langle n+k|\hat{\mathcal{O}}^{cm}|n+k\rangle\Big)\rho^{cm}_{nn}(0)\,. \tag{12}$$

As in the previously discussed decoherence effect on $P_{\downarrow}(t)$, only diagonal contributions of $\hat{\rho}^{cm}(0)$ enter Eq. (11), whereas the existence of the damping for the position $\hat{x}^{cm}$ (as well for any other negative-parity observable) crucially rests on the coherence of the initial state number occupation. Relation (12) allows one to evaluate the asymptotic position variance: by choosing $\hat{\mathcal{O}}^{cm} = (\hat{x}^{cm} - \langle\hat{x}^{cm}\rangle)^2$, one gets

$$\lim_{t\to\infty}\Delta\hat{x}^2_{cm}(t) = 4\left(\frac{\hbar}{2m\omega}\right)\left(\overline{n} + \frac{1}{2} + \frac{k}{2}\right), \tag{13}$$

with $\overline{n} = \sum_{n=0}^{\infty} n\,\rho^{cm}_{nn}(0)$ the initial average vibrational quantum number. Hence, the position damping (10) does not imply a parallel suppression of the position variance. Analogous considerations can be repeated for the center-of-mass momentum, enforcing the validity of the Heisenberg principle. Moreover, center-of-mass position, momentum and energy are linked together via a simple asymptotic relationship,

$$\lim_{t\to\infty}\left\langle\frac{\hat{H}^{cm}(t)}{2}\right\rangle = \lim_{t\to\infty}\left\langle\frac{m\omega^2\hat{x}^2_{cm}(t)}{2}\right\rangle = \lim_{t\to\infty}\left\langle\frac{\hat{p}^2_{cm}(t)}{2m}\right\rangle, \tag{14}$$

pointing out that energy equipartition is preserved when quantum damping is in action. Eq. (11) deserves some further comments. First, the asymptotic average energy corresponds to heating of the vibrational motion with respect to the initial value $\langle\hat{H}^{cm}(0)\rangle = \hbar\omega(\overline{n}+1/2)$, which is reasonable due to the blue sideband driving. In the case $k = 0$ however, the energy is conserved for *any* value of the measurement coupling κ, as physically understandable since the laser is tuned to the electronic transition and the interaction does not change the number of vibrational quanta. Therefore, no back-action of the measuring apparatus on the motional energy is present in such conditions, consistently with

the recent analysis by de Matos Filho and Vogel where quantum nondemolition measurements of the vibronic energy based on a zero-quantum JCM interaction have been proposed [21].

Let us conclude by briefly discussing the observability of the effects predicted on both the internal and external motions of the ion. As a preliminary remark, the ensemble formalism throughout adopted here finds its justification since averages of measurements on single ions each time identically prepared are to be handled in laboratory. Concerning the deformation of the two-level dynamics (7), one has essentially to design a more elaborated version of the experiment performed by Itano *et al.* to observe the quantum Zeno effect [22]. Compared to this, a complication arises due to the dependence of the critical measurement coupling κ^{crit}_{nn+k} on the Jaynes-Cummings manifold, owing to which a sharp transition from oscillatory to overdamped regime is only expected in the simplest case of a Fock state (9). An experimental scheme aimed at testing quantum damping is slightly more complicated since a further measurement is required to check the predicted average values. This is not conceptually different from the verification of any quantum mechanical prediction. If, for instance, positional damping is considered, the procedure involves many replicas of the evolution of the whole system + meter up to time t each followed by an instantaneous measurement of the ion position. By subsequently repeating the experiment for different times t, the average value $\langle x^{cm}(t)\rangle$ can be accumulated and compared to (10). In practice, the readout of the ion position could be obtained by measuring the charge induced on two auxiliary end caps in the trap, provided the signal-to-noise ratio is large enough [23]. Alternatively, less direct methods could be adopted, such as the already demonstrated selective measurement of position in which fluorescence light induced by a localized probe beam is detected [24]. Leaving aside to a future analysis the detailed discussion of the detection scheme, a more quantitative insight of the relevant timescales can be already gained by assuming for the parameter κ the same value adopted in [13] to fit the data of the quantum Zeno experiment [22], $\kappa \approx 4.9 \cdot 10^4\,\mathrm{s}^{-1}$. Also notice that the same auxiliary transition ${}^2S_{1/2} \rightarrow {}^2P_{3/2}$ of ${}^9\mathrm{Be}^+$ has been chosen in [8]. The resulting decay time is $\tau = 4\kappa^{-1} = 816\,\mu\mathrm{s}$ for both the internal and motional dynamics, to be compared with the decoherence reported in [8] for a Fock state, namely $\tau^{exp} = 84\,\mu\mathrm{s}$ for the $|\downarrow, 0\rangle \rightarrow |\uparrow, 1\rangle$ transition. Being additional relaxation mechanisms due to spontaneous decay of the electronic transition and mechanical dissipation of the ion motion negligible [22,23], these numerical estimates show that a hundredfold improvement in the stability performances of the experimental setup should make the overall decoherence dominated by the measurement induced one. Since a ratio $\kappa/\kappa^{crit}_{01} = 2.1 \cdot 10^{-2}$ is obtained in the example considered above, both a Rabi-like behavior in the two-level dynamics and an underdamped oscillation in the vibronic motion are expected. An increase of this ratio can be attained by lowering the fundamental Rabi frequency Ω_0 by two orders of magnitude, allowing one to enter the region where the two contributions in the right-hand side of Eq. (1) strongly compete.

In summary, we have proposed a quantitative model to examine the influence of a continuous measurement process on the dynamics of a trapped ion. The decoherence induced by the interaction with the measuring apparatus has been identified through peculiar modifications of both electronic and vibrational observable properties of the ion. Detecting such effects gives further stimulus to deepen our knowledge of the mesoscopic world and may complement the direct reconstruction of the density matrix via quantum tomography of single trapped ions recently achieved in laboratory [25].

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